Rest Break Policy Comparison for Heavy Vehicle Drivers in Australia

Saman Eskandarzadeh*[1] and Behnam Fahimnia†[2]

[1,2]Institute of Transport and Logistics Studies, University of Sydney, NSW 2006, Australia

Abstract

Carriers and postal companies are under increasing pressure to reduce their operating costs and increase efficiency. One way to reduce costs is to improve the utilisation of drivers' working hours by employing more efficient rest break policies. A rest break policy is a restrictive set of rules consistent with national regulations for hours of service. We develop and validate a novel framework to model and analyse a class of these policies that concern the location of the rest breaks. In particular, we compare two representative rest break policies using data from a major Australian postal carrier. The first policy imposes no restriction on the location of a rest break. The second policy requires the driver to return to a depot for rest taking allowing time for socialising and making use of full amenities. Using postal transport data from Sydney metropolitan area, we find that the difference between the two polices in terms of tour length is just over 1%. We further apply the proposed framework to assess the impact of increasing the minimum break time on the two representative policies.

1 Introduction

Carriers and postal companies have been under increasing pressure to reduce prices and increase their service levels, often measured in form of delivery times (PwC (2016), Briest et al. (2019)). They also feel obligated to their customers and the next generations to reduce their carbon footprint [1] through innovation and undertaking sustainable initiatives (Fahimnia et al. (2015)). One approach to tackle this pressure is to become more productive and cost-efficient through efficient planning and optimisation of pickup and delivery operations (Briest et al. (2019)). The primary service of carriers is to collect customer products (mail or parcels) and deliver them to given destinations/customers. If shipping volume is much less than a truckload, direct shipping may be too cost inefficient; in which case, the products destined to the same region are consolidated to utilise the economies of scale.

Large carriers have multiple consolidation centres allowing them to coordinate the product flow between these centres and between the centres and the customers, in order to reduce costs and maintain/improve service levels. An effective robust scheduling is essential to plan the resourcing and timing of the pickup and delivery jobs. This is a challenging problem, the classic form of which is known as Pickup and Delivery Problem with Time Windows, commonly referred to as PDPTW (Ropke and Cordeau (2009); Dumas et al. (1991)).

A missing element in a PDPTW problem is the planning of the rest breaks. Most countries have certain rest break regulations for truck drivers. Australia (NHVR (2020)), Canada (Justice Laws (2020)), European Union (EU) countries (Europa 1 (2020); Europa 2 (2020)), and the US (Federal Register (2020)) have applied

*saman.eskandarzadeh@sydney.edu.au

†ben.fahimnia@sydney.edu.au

[1]https://sustainability.ups.com/sustainability-strategy/environmental-responsibility

strict rest break regulations to reduce the risk of accidents due to prolonged hours of work with insufficient rest. The rest break regulations in Australia are quite unique with less structural resemblance to what is practised in the rest of the world. There are multiple regulatory frameworks for rest breaks, but the most commonly used framework is based on the standard hours regulations for solo truck drivers. We refer to this as SH framework/rules[2 3].The regulations apply to heavy vehicles with Gross Mass larger than 4.5 tonnes, excluding rolling stocks such as trains. Our focus in this paper is on planning for short Hours of Service (HOS) (i.e., less than 13 hours service per day) for heavy truck drivers which is applicable to carrier operations in major cities and many regional areas. We interchangeably use the terms 'heavy truck' or 'truck' in the paper. According to the SH framework, for the short HOS, each truck driver cannot work continuously more than 5:15, 7:30, and 10 hours without taking break(s) of at least 15, 30 and 60 minutes, respectively (in 15-minute resting blocks).

Truck drivers in cities and regional areas complete multiple pickup and delivery jobs in every shift. Finding an optimal schedule with respect to pickup and delivery time windows while also considering the SH rules is a formidable challenge. The existing methodologies either lack the flexibility to easily accommodate carrier specific preferences or fail to utilise the full flexibility provided by the SH rules. As a result, the real applications of these methodologies are rather limited, especially when it comes to planning for the short HOS.

To illustrate the cost implications of break rules, we introduce the following example.

Example 1. There are three customers to be visited by a truck driver for daily product collection. The collected products are delivered to the depot. Service time at each stop is 20 minutes. Customers 1, 2, and 3 have to be visited within time windows $[100, 260]$, $[0, 1440]$, and $[100, 420]$, respectively; where $[a, b]$ denotes the time window with earliest start time a and latest start time b for the visit. Times a and b are measured in minutes passed from 6:00 AM. The distance matrix (in minutes) between customer locations and the depot (with index 0) is as follows:

$$D = \begin{bmatrix} 0 & 96 & 112 & 136 \\ 96 & 0 & 120 & 144 \\ 112 & 120 & 0 & 232 \\ 136 & 144 & 232 & 0 \end{bmatrix}$$

Without breaks, the optimal sequence of visits is 0-2-1-3-0 with a total duration of 592 minutes. When breaks are taken into consideration, this sequence is not feasible anymore. The reason is that it takes 416 minutes for a truck to get to Customer 3 and since according to the SH rules, a break of 15 minutes is required before servicing Customer 3, the truck can only start servicing Customer 3 at 431 minutes at earliest, which is outside the acceptable time window for Customer 3. Under the SH framework, an optimal tour is 0-1-3-2-0 with a duration of 724 minutes with two 30-minute breaks at Customer 2 and Customer 3. So, if the break times are excluded, the work time is increased from 592 minutes to 663 minutes from the first optimal sequence to the second optimal sequence. This is a significant increase of 12% in the total work time.

Our research was motivated by an optimisation/scheduling problem facing a major postal carrier in Australia. Typical daily operations of the proposed company are as follows. Mail and parcels are picked up from collection facilities and bulk customers. They are then shipped to middle sortation facilities and subsequently to delivery centres. From delivery centres, they are carried to destination points. The shipping between bulk customers and facilities is mainly handled by trucks. The aim is to maximise the truck utilisation. We study this problem for a single vehicle and a single driver operating under the Australian the HOS regulations. We are given a set of pickup and delivery jobs where each job requires some parcels to be picked up from one location and delivered to another location. The pickup as well as the delivery should be done within pre-specified time windows. Consistent with practice, demand is measured in cubic meters. For heavy items, more space is

[2] `https://www.nhvr.gov.au/safety-accreditation-compliance/fatigue-management/work-and-rest-requirements/standard-hours`

[3] `https://www.nhvr.gov.au/safety-accreditation-compliance/fatigue-management/counting-time`

allocated in the truck to keep the load balance of the truck. Therefore, weight can be translated into cubic meters. We aim to schedule all jobs in a single tour while respecting the SH break rules.

The SH rules do not require the drivers to return to the depot for taking breaks. However, some logistic providers, including the postal company that motivated this study, schedule the tours in a way that the drivers end up taking their breaks in the depot which is fully equipped for rest taking and also allows the drivers to socialise. In our analysis, we provide a cost comparison of the two policies.

The problem will be formulated using Mixed-Integer Programming (MIP) which is known to be a suitable methodology for tactical and operational decision-making problems in logistics and supply chain management. This is evidenced by numerous commercial solvers and academic papers published on both MIP theory and applications (Dong et al. (2020); Zhen et al. (2020); Li et al. (2020); Schiffer et al. (2019)). We develop an exact MIP model to solve a single-vehicle PDPTW under the SH rules. We model the problem under the SH rules with (scenario 1) and without (scenario 1) restriction for break location. A "unified" methodology will then be developed to compare the two scenarios in terms of the tour length. We refer to this as a unified approach/methodology since both scenarios are modelled in the same fashion. Finally, we use the models and the methodology on a real dataset provided by a major postal carrier.

The scheduling of rest breaks given a sequence of tasks for a single truck and a single driver is referred to as Truck Driver Scheduling Problem (TDSP). Our contribution to the literature of the TDSP is threefold. (1) We pioneer the development of an MIP model that allows for a flexible break location (i.e., a break may take place between tasks at any location, not just at the depot). The model is tractable by existing commercial optimisation packages. MIP models are much easier to apply or extend compared to other exact approaches such as dynamic programming which require customisation and is more time-consuming to implement. For dynamic programming, no generic solver exists in the market; while there are numerous commercial and open-source packages to solve MIP models. (2) We develop a unified methodology for modelling various restrictions on break locations. (3) We use real data to validate the proposed model and methodology and compare representative rest break policies.

The rest of the paper is organised as follows. Section 2 presents a review of the literature on rest break optimisation under the HOS constraints and related policies. A formal description of the problem under investigation is presented in Section 3. Section 4 presents a set of necessary and sufficient conditions for tour feasibility under the SH rules. These conditions will then enable us to develop an exact MIP model that can be customised to formulate and evaluate different policies. Section 5 compares the impact of two rest break scenarios on a real dataset obtained from a major postal carrier in Australia. We also estimate the price of each scenario compared to a situation with no rest break. Finally, Section 6 presents a summary of the key findings as well as directions for future work in this domain.

2 Literature review

There are two streams of research relevant to this study. The first stream considers the HOS regulations in scheduling of truck deliveries in given time windows. The classic problem in this stream is the TDSP. In the TDSP problem, there is a single vehicle, and the sequence of tasks is given. Therefore, the sequence, which affects the total driving times, is not a decision variable. In this stream of research, there are also problems in which the TDSP is integrated with Vehicle Routing Problem (VRP). The focus is more on developing solution methods or ideas to help improve operational scheduling/planning. The second stream of research, on the other hand, aims to study the TDSP or its variants from tactical planning and/or policymaking perspectives.

Our review of the first stream of literature starts by the work of Xu et al. (2003), the first study to integrate the TDSP with Pickup and Delivery Problem under the HOS regulations in the US. A column generation based heuristic algorithm is presented as a solution method. Archetti and Savelsbergh (2009) study the TDSP under the US HOS regulations and propose a polynomial time algorithm that either finds a feasible solution for the problem with minimum total rest time or establishes infeasibility. In their setting, time windows are only defined for pickup tasks.

Goel (2009) and Ceselli et al. (2009) were the first to study the TDSP under the EU HOS regulations. Both papers solve the TDSP integrated with routing decisions using heuristic algorithms. The problem setting in Ceselli et al. (2009) was used in a decision support system of a transport company in Italy. Similar to our setting, they consider a short less-than-one-day time horizon. Goel (2010) develops the first exact algorithm for the TDSP under the EU regulations. Kok et al. (2010) and Prescott-Gagnon et al. (2010) develop heuristic algorithms for the integrated TDSP with routing under the EU HOS regulations.

Goel (2012b) is the closest paper to our work and the first study that suggests an MIP model for the TDSP under the Australian HOS regulations. Our work differs from the work of Goel in four dimensions. (1) We relax the assumption that the sequence of tasks is known; therefore, we also incorporate routing decisions into our model. (2) We schedule tasks for a single tour for short HOS (less than 13 hours). In our application in Sydney Metropolitan area, the maximum shift time for drivers is 12 hours which seems to be a standard characteristic of city transportation jobs around the globe. In Goel's model the focus is on a longer time horizon over one week which usually applies to intercity transport. (3) Our primary aim is to develop a framework for evaluation of the HOS regulations in practice, not a methodology for generating schedules. Nevertheless, we show that the MIP models presented in this paper perform very well on real data and, in most instances, generate optimal tours in less than one minute. (4) We also relax some of the other assumptions that may not replicate the reality. Specifically, Goel (2012b) assumes that rest breaks can only be taken immediately after arrival at a location and before starting to work at that location. It also assumes all time values are multiples of 15 minutes. These two critical assumptions make the resulting models too restrictive when compared to real scenarios. In our study, we relax these two restrictions.

In another study, Goel et al. (2012) investigate the TDSP under the Australian HOS regulations using a dynamic programming approach. An exact dynamic programming algorithm and a set of heuristics are presented to find a feasible solution. If no feasible solution exists, the algorithm reports infeasibility. Although they do not restrict the resting locations, all time values are multiples of 15 minutes. In the same year, Goel (2012a) introduces a generic MIP and dynamic programming approach for solving the TDSP under the EU and US regulations. The MIP model imposes restriction on rest break locations, but the dynamic programming relaxes this assumption. Goel and Rousseau (2012) introduce an approach for solving the TDSP under Canadian regulations. It presents an exact algorithm for either finding a feasible solution or proving infeasibility. Sartori et al. (2021) consider TDSP problem under the EU HOS regulations. They assume there is a precedence relationship between tasks. They also assume that the tasks in each tour are predetermined. The study proposes an algorithm with exponential worst-case runtime which finds a feasible task schedule for each tour.

The second stream of research is not as mature and established. Goel and Vidal (2014) propose a metaheuristic to compare the HOS regulations in EU, Canada, Australia and the US in terms of accident risk and operating costs, considering total distance and fleet sizes. They compare these regulations on instances with 100 customers for a planning horizon of 144 hours. The average time window in their study is 7 hours which is quite large compared to urban services where the average time window is less than an hour. For express post, the time windows are even shorter. They use a modified version of the heuristic algorithm introduced by Goel et al. (2012) for assessing compliance with the Australian regulations. The original algorithm assumes all time values are multiples of 15 minutes which restricts the application of this approach. This assumption is rather relaxed in the modified algorithm. However, in the solutions generated by the modified algorithm, the start and end times of all off-duty periods are still in multiples of 15 minutes. This leads to allocation of redundant times to a tour which is particularly not desirable for the short HOS in which 15 minutes can be allocated to a stand-alone piece of work. Our model is free of this restriction.

Goel (2014) assesses the impact of the new HOS regulations in the US (changed in 2013) on the operating costs of transport companies using a simulation-based methodology, initially proposed by Goel and Vidal

Table 1: Summary of Studies on Truck Driver Scheduling Problem

Article	Year	Problem	HOS Regulations			
			AUS	CAN	EU	US
Xu et al. (2003)	2003	TDSP+Routing				✓
Archetti and Savelsbergh (2009)	2009	TDSP				✓
Goel (2009)	2009	TDSP+Routing			✓	
Ceselli et al. (2009)	2009	TDSP+Routing			✓	
Goel (2010)	2010	TDSP			✓	
Kok et al. (2010), Prescott-Gagnon et al. (2010)	2010	TDSP+Routing			✓	
Goel (2012b), Goel et al. (2012)	2012	TDSP	✓			
Goel (2012a)	2012	TDSP			✓	✓
Goel and Rousseau (2012)	2012	TDSP		✓		
Rancourt et al. (2013)	2013	TDSP+Routing				✓
Goel (2014)	2014	TDSP+Routing				✓
Goel and Vidal (2014)	2014	TDSP+Routing	✓	✓	✓	✓
Koç et al. (2016)	2016	TDSP				✓
Goel and Irnich (2017)	2017	TDSP+Routing			✓	✓
Koç et al. (2017)	2017	TDSP+Routing				✓
Tilk and Goel (2020)	2020	TDSP+Routing			✓	✓
Sartori et al. (2021)	2021	TDSP+Routing			✓	

(2014). The study uses monetised accident risk, and time-based and distance-based costs as performance metrics. In another study, Koç et al. (2017) investigates the TDSP with a rich objective function that takes into account the cost of engine idling. The authors develop an MIP model considering the HOS regulations in the US to assess the impact of engine idling and its policy implications.

Indeed, the second stream of research focuses primarily on operating and safety measures in analysing the HOS regulations. Research in this area overlooks the implementation challenges and the direction/intensity of the impacts of such regulations. It is our intention in this paper to explore this topic from the perspective of rest break location. In addition, previous studies have often developed hard-to-use, relatively inflexible, and highly specialised methodologies to tackle the related problems. It is not possible to conveniently accommodate various criteria or preferences in such complex models. We will address this issue by developing a simple and highly flexible MIP-based methodology which is particularly useful in real world situations where rapid decision making is essential. Table 1 provides a comprehensive summary of studies on the TDSP and its variants.

3 Problem Description

We are given a set of tasks $Q = \{0, \cdots, 2n+1\}$. Task $i \in Q$ has earliest start time t_i^e, latest start time t_i^l, location l_i, pickup quantity q_i, and service time s_i. At each location l, there is a fixed preparation time prior to the commencement of the service, denoted by s_l^1. After the completion of the service, there is a fixed preparation time denoted by s_l^2. Preparation times could account for activities such as parking and unparking or loading and unloading. Since multiple tasks could be scheduled consecutively at one location, the preparation times cannot be incorporated into the task service time; hence, we define them separately. For each pickup task i there is an associated delivery task $n+i$. Let set $P = \{1, \cdots, n\}$ and set $D = \{n+1, \cdots, 2n\}$ be the set of pickup and the set of delivery tasks, respectively. Let also tasks 0 and $2n+1$ denote the start and the finish task at the depot with zero pickup quantities, respectively. We define a job as a pair of a pickup and a delivery tasks. For $i \in P$, job $(i, n+i)$ is to pick up quantity q_i from location l_i and deliver it to location l_{n+i}. Let set $O = \{(i, n+i) : i \in P\}$ be the set of all jobs. Since q_i by definition refers to pickup quantity, we denote the pickup quantity of a delivery task $n+i$ by the negative of q_i, i.e., $q_{n+i} = -q_i$. A tour is defined as a sequence of all tasks where task $i \in Q$ starts at time x_i^s. Each tour is serviced by a truck with capacity c. We denote the maximum tour time by t_{max}. The time distance between locations of task i and task j is denoted by d_{ij}. The problem is to find a minimum length tour such that all pickup and delivery tasks are serviced within their time windows and the HOS regulations are respected. Without considering the HOS regulations, this is a classic vehicle routing problem with pickup and delivery time windows.

3.1 Break time

There are three established regulatory frameworks in Australia including Standard Hours (SH), Basic Fatigue Management (BFM), and Advanced Fatigue Management (AFM). These frameworks are structurally identical. In this paper, we merely focus on the SH regulations which is the most broadly adopted framework in industry. The BFM and AFM regulations allow for longer hours of work, but are only applicable to accredited transport operators. In any case, within cities and regional areas, the working hours are often limited to maximum 12 hours. Considering the short trips and the short HOS, we do not need to incorporate restrictions on night shifts as well as the rules requiring drivers to take long rest breaks out of the vehicle or inside the vehicle in certain conditions [4]. Such rules and restrictions are usually taken into account by planners/schedulers for rostering and task assignment purposes. We consider this outside the scope of our analysis.

According to the SH rules, the minimum break is 15 minutes with no restriction on the location of the break. There are however Australian carriers who prefer the breaks to be taken at the depot, if possible. We are interested to know how this alternative practice compares financially to the the standard SH rules. Since the length of a tour is the primary cost driver for that tour, we compare these alternative policies using the length of the tours as the primary measure. We specifically compare two policies: policy Standard Hours (SH) and policy Standard Hours at Depot (SHD). In the SH policy, all breaks are scheduled according to the SH rules. In the SHD policy, the breaks are still scheduled according to the SH rules taking into account an extra restriction that all breaks need to be scheduled at the depot. This restriction of the SHD policy may incur a significant cost as illustrated in the following examples.

Example 2. We have 3 customers with some parcels for collection. All parcels need to be transported to the depot. Service duration is 20 minutes at each location irrespective of the number of parcels loaded or unloaded. We set the index of the depot to 0. The distance matrix in minutes is equal to:

$$D = \begin{bmatrix} 0 & 90 & 90 & 120 \\ 90 & 0 & 10 & 60 \\ 90 & 10 & 0 & 50 \\ 120 & 60 & 50 & 0 \end{bmatrix}$$

We assume that the truck allocated to service the customers has enough capacity to complete all tasks without the need to go back to the depot in the middle of the tour. For the SH policy, an optimal tour is $0-1-2-3-0$ with a duration of 365 minutes and a 15-minute break at customer 3 location. However, an optimal tour for the SHD policy would be $0-3-2-0-1-0$ with a duration of 570 minutes and a break of 30 minutes at the depot given that the truck must visit the depot to take a break in the middle of the tour. Not only SHD policy caused a significant increase in tour duration, but it also affected the order of the visits. The SHD policy made the tour long enough to need a 30-minute break, instead of 15 minutes as required by the SH rules. We note that since the break location is not restricted in the SH policy, the only way that the SH policy could change

[4] `https://www.nhvr.gov.au/safety-accreditation-compliance/fatigue-management/work-and-rest-requirements/standard-hours`

the optimal sequence of visits to customers is when strict time windows are applied. In order to ensure that a sequence is compliant with the SH policy, we just need to schedule the breaks at appropriate times. Adding breaks does not require us to change the sequence, nor does it increase the total tour length excluding the total break time.

Example 3. Consider the data in Example 2 with the following distance matrix:

$$D = \begin{bmatrix} 0 & 96 & 112 & 136 \\ 96 & 0 & 120 & 144 \\ 112 & 120 & 0 & 232 \\ 136 & 144 & 232 & 0 \end{bmatrix}$$

Customer 3 has to be visited within time window $[100, 420]$ and Customer 1 has to visited within time window $[100, 260]$. If we ignore both policies, optimal tour is $0-2-1-3-0$ with a duration of 592 minutes. Under the SH policy, this tour is not feasible as it would take 416 minutes for the truck to get to Customer 3 without break and since a break of 15 minutes is required before getting to Customer 3, the truck could not start servicing Customer 3 before 431 min – which is obviously outside the acceptable timeframe. Under the SH policy, an optimal tour is $0-1-3-2-0$ with a duration of 724 minutes, a 30-minute break at Customer 3, and a 30-minute break at Customer 2.

The SHD policy has two main advantages over the SH policy. First, it is more convenient for the drivers since they can socialise with other drivers and use the amenities available at the depot. Second, it is easier to plan for compliance with the HOS regulations using the SHD policy as the choices are less compared to the SH policy. However, the associated logistics cost could be excessively high; hence a thorough cost/benefit analysis is essential.

In all these policies, we assume that the break times cannot happen during service times. This is consistent with the current postal service practice, since a service at a pickup and delivery location cannot be interrupted by a break. There are however certain activities that could be interrupted by a break (e.g., driving times). Regardless of the rest break policy, we assume in all our models and experiments that service times at the pickup and delivery locations cannot be interrupted.

4 The Models

In this section, we present an MIP model for each policy. Before that, we prove an important theorem which gives us both necessary and sufficient conditions for compliance of a given tour. We can think of a tour as a sequence of two types of periods. We call them flexible and inflexible periods. A flexible period has two features. First, it can be all work time or all break time. Second, if it is a mixed work-break period, the break(s) can start at any time during the period. In contrast, an inflexible period has no break and consists of a sequence of work blocks that satisfy two conditions: (a) none of the work blocks was allowed to be interrupted with a break, and (b) no break was allowed to be scheduled between the work blocks. In general,in any given tour, we first find inflexible periods. The time period between any two consecutive inflexible period is a flexible period. These concepts simplify our analysis as we can equally treat the contents of inflexible periods, and can consider every inflexible period as a single solid work block. Later in this section, we prove Theorem 1 that tells us, for a given tour, if there are enough break blocks inside flexible periods, then there exists at least a feasible solution with respect to the SH rules. A feasible schedule can be found by shifting break blocks inside the flexible periods. This shifting of break blocks does not change the length of the tour or the schedule of tasks. Since our objective is to find the optimal tour duration, we do not mind how the breaks are exactly scheduled.

Let $[n] = \{1, \cdots, n\}$. For inflexible period i, we denote its start time and end time by s_i and f_i , respectively. We denote a tour with n inflexible periods i by $\{(s_i, f_i)\}_{[n]}$ where $f_i \leqslant s_{i+1}$ for all $i \in [n-1]$. All the periods between consecutive inflexible periods are flexible periods. We assume that the period ending at s_1, and the period starting after f_n are rest breaks. An SH-type break rule can be defined by a positive and real parameter a and two positive integers b and δ. We refer to an SH-type rule by triple (a, b, δ). In any period of length a minutes, there should be at least b break blocks of length δ minutes each. For example, for the first SH break rule $a = 330$, $b = 1$, and $\delta = 15$. We first prove a general theorem for a single SH-type rule which sets necessary conditions for feasibility of a given tour $\{(s_i, f_i)\}_{[n]}$. Consider tour $\{(s_i, f_i)\}_{[n]}$, and SH-type rule (a, b, δ) with b_i breaks in flexible period $[f_i, s_{i+1}]$ for all $i \in [n-1]$.

Lemma 1. *If there exists an SH-feasible schedule of breaks with b_i breaks in each flexible period i then for all $i, j \in [n]$ where $i \leqslant j$, and for all $r \in \{0, \cdots, b-1\}$, we have $\sum_{k=i}^{j-1} b_k \geqslant b - r$ if*

$$f_j - s_i > a - (2 + r)\delta. \tag{1}$$

Proof. For $i, j \in [n]$ and $r \in \{0, \cdots, b-1\}$, consider interval $[s_i - \delta + \epsilon/2, f_j + (r+1)\delta - \epsilon/2]$ for a small positive and real valued number ϵ. Note that this interval contains interval $[s_i, f_j]$ for small ϵ. If condition 1 holds, then the length of the interval is bigger than $a - \epsilon$. For sufficiently small ϵ, the length of interval is equal to or greater than a. Therefore, it requires at least b break blocks according to the SH rules. Since ϵ is a positive number, at most r break blocks can be placed outside of interval $[s_i, f_j]$ and inside of interval $[s_i - \delta + \epsilon/2, f_j + (r+1)\delta - \epsilon/2]$. It follows that at least $b - r$ break blocks should be inside the interval $[s_i, f_j]$. □

If the times and parameters are all integers then condition 1 boils down to

$$f_j - s_i \geqslant a - (2 + r)\delta + 1. \tag{2}$$

Now, consider three SH rules $(330, 1, 15)$, $(480, 2, 15)$, and $(660, 4, 15)$. We assume that the length of the tour, i.e., $f_n - s_1$, does not exceed 13 hours or equivalently 780 minutes. We further assume all times are integer values.

Theorem 1. *There exists an SH-feasible schedule of breaks if and only if for all $i, j \in [n], i \leqslant j$, we have:*

1. *if $f_j - s_i \geqslant 301$, then $\sum_{l=i}^{j-1} b_i \geqslant 1$,*
2. *if $f_j - s_i \geqslant 451$, then $\sum_{l=i}^{j-1} b_i \geqslant 2$,*
3. *if $f_j - s_i \geqslant 616$, then $\sum_{l=i}^{j-1} b_i \geqslant 3$,*
4. *if $f_j - s_i \geqslant 631$, then $\sum_{l=i}^{j-1} b_i \geqslant 4$.*

Furthermore, this feasible schedule can be obtained by just scheduling b_i break block(s) within flexible period $[f_i, s_{i+1}]$ comprised of $b_i\delta$ break time and $s_{i+1} - b_i\delta - f_i$ combined preemptive work time and idle time for every $i \in [n-1]$.

Proof. Refer to Appendix A. □

Given the objective of tour length minimisation, the immediate consequence of the above theorem is that it allows us to explicitly schedule jobs without the need to schedule breaks. Therefore, in the first stage, we find an optimal schedule of jobs. Then, in the second stage, we can find an explicit schedule of breaks without affecting the optimality or the scheduled start time of the jobs.

Under both policies, all tours should start and end at the depot. We denote the start location and the end location of a tour by 0 and $n + 1$, respectively. Since there is only one depot, the start and end locations are identical. Let us define some notations (refer to Table 2 for primary notations and all notations not defined within the text). We denote the set of all possible pairs of consecutive tasks by H which is defined as $\{(i,j) : i \in P \cup D \cup \{0\}, j \in P \cup D \cup \{2n+1\}\}$. In real applications, some of these pairs might not be acceptable due to operational, regulatory, or contractual requirements. Let $A \subseteq H$ be the set of acceptable pairs. The objective function is to minimise the length of the tour; that is,

$$\min x^s_{2n+1} - x^s_0. \tag{3}$$

Table 2: Notations

Symbol	Definition
$[n]$	$\{1, \cdots, n\}$, n is a positive integer
Q	set of all tasks, i.e., $\{1, \cdots, 2n+1\}$
P	set of pickup tasks, i.e., $\{1, \cdots, n\}$
D	set of delivery tasks, i.e., $\{n+1, \cdots, 2n\}$
O	set of all jobs $(i, n+i)$ where i is a pickup task and $n+i$ is a delivery task
H	set of all possible pairs of consecutive tasks, or equivalently $\{(i,j) : i \in P \cup D \cup \{0\}, j \in P \cup D \cup \{2n+1\}\}$
A	a given subset of H
$\mathcal{B}$	$\{15k : k \in [k_{max}]\}$
k_{max}	the maximum number of break blocks required between any two consecutive tasks under the SH rules
t_{max}	maximum tour time, a number less than 780 minute or 13 hours
l_i	location of task i
t^e_i, t^l_i	earliest start time and latest start time of task i
s_i	service time specific to task i
s^1_l	preparation time specific to location l before service starts
s^2_l	preparation time specific to location l after service ends
q_i	quantity of task i for collection (if task i is delivery, q_i is negative)
d_{ij}	distance in time between location of task i and j
c	capacity of a truck
x_{ij}	value 1 indicates task j is immediately after task i
x^s_i	service start time for task i
x^B_i	accumulated break time by the start time of task i
x^c_i	vehicle's available capacity at the start time of task i
y^b_{ij}	value 1 indicates that there should be b minutes break between finish time of task i and start time of task j
x^b_{ij}	value 1 indicates there is b minutes break between finish time of task i and start time of next task j

All the given tasks must be completed in a single tour. To make sure we have a sequence of tasks in the solution, each task $i \in P \cup D$ can be succeeded and preceded by only one task. Let us name the set of constraints for modelling of this requirement as "service constraints".

Service constraints:

$$\sum_{(i,j)\in A} x_{ij} = 1 \qquad \text{for all } i \in P \cup D \cup \{0\}, \tag{4}$$

$$\sum_{(j,i)\in A} x_{ji} = 1 \qquad \text{for all } i \in P \cup D \cup \{2n+1\}, \tag{5}$$

The capacity of a truck is c cubic meters. After each collection i, the available capacity drops by q_i; and after each delivery j, the available capacity is increased by $-q_j$.

Capacity constraints:

$$x^c_j \leqslant x^c_i - q_i + M(1 - x_{ij}) \qquad \text{for all } (i,j) \in A, \tag{6}$$

$$0 \leqslant x^c_i \leqslant c \qquad \text{for all } i \in P \cup D \cup \{2n+1\}, \tag{7}$$

$$x^c_0 = c. \tag{8}$$

M is a number bigger than c. Constraint 6 ensures that the available capacity of the truck at the start of servicing task j is not more than the available capacity at the end of the immediate preceding task i, that is, $x^c_i - q_i$.

Time constraints:

$$t^e_i \leqslant x^s_i \leqslant t^l_i \qquad \text{for all } i \in P \cup D \cup \{0, 2n+1\}, \tag{9}$$

$$x^s_{2n+1} + s_{2n+1} + s^1_{l_{2n+1}} - (x^s_0 + s^2_{l_0}) \leqslant t_{max}. \tag{10}$$

Constraints 9 ensures that the time window for each collection or delivery is respected. Constraint 10 ensures that the length of the tour does not exceed the maximum tour length t_{max}.

The following sub-sections present the constraints specific to each policy. The break constraints are the direct consequence of Theorem 1.

4.1 The SH policy

If we look at each pair of tasks (i,j) in $A \backslash O$, there are four possibilities in a feasible solution.

1. Task j is the next task after i,
2. Task i is the next task after j,
3. Task j is not the next task after i but succeed task i,
4. Task i is not the next task after j but succeed task j.

For jobs $(i,j) \in O$, the order is predetermined. The order of all other tasks is decided by the model. We name the constraints for the modelling of this requirement as "precedence constraints". Let set $\mathcal{B} = \{15k : k \in [k_{max}]\}$, where k_{max} is the maximum number of break blocks required between tasks i and j in any feasible solution with respect to the SH rules. Following Theorem 1 and knowing that tasks i and j are consecutive tasks, it is obvious that the maximum number of breaks is 4.

Let s_{ij} denote the total service time and preparation time between the start of task i and the start of next task j. The total service time at each location is the total service time of consecutive tasks done at the location plus the total preparation time. The preparation time before the first task at a location can include vehicle parking and any other preparation that is needed for collection/delivery. The preparation time after the last task at a location can include any activity that is needed prior to departure including packing up and unparking.

$$s_{ij} = \begin{cases} s_i & \text{if } \; l_i = l_j, \\ s_i + s^2_{l_i} + s^1_{l_j} & \text{otherwise.} \end{cases}$$

Precedence constraints:

$$x^s_i + s_{ij} + d_{ij} + \sum_{b\in\mathcal{B}} bx^b_{ij} \leqslant x^s_j + M(1 - x_{ij}) \qquad \text{for all } (i,j) \in A\backslash O, \tag{11}$$

$$x^s_i + s_{i,n+i} + d_{i,n+i} + \sum_{b\in\mathcal{B}} bx^b_{i,n+i} \leqslant x^s_{n+i} \qquad \text{for all } (i, n+i) \in O. \tag{12}$$

M denotes a sufficiently big number. For precedence constraint, $M \geqslant T$ is sufficient. Consider pair of tasks $(i,j) \in A\backslash O$. Constraint 11 enforces possibilities 1 or 2 by the value of binary variable x_{ij}. If i and j are not consecutive, then Constraint 11 becomes inactive for (i,j). However, by service constraints there is a set of tasks that come between i and j. Constraint 11 indirectly enforces possibilities 3 and 4 in this case for (i,j) by directly enforcing possibilities 1 and 2 on all pairs of consecutive tasks from i to j. For $(i,j) \in O$, Constraint 12 imposes that task j should succeed task i because it is a delivery task. For job (i,j) only possibilities 1 and 3 can happen in a feasible solution. The other two possibilities are ruled out by Constraint 12. These two possibilities are enforced by combining the service constraint and the precedence constraints.

We assume that the drivers cannot take a break once the preparation time starts in a location until the after-task preparation ends. This assumption is in line with our postal service application (and with general logistics practice for short service times) and is consistent with the previous literature in this domain (e.g., Goel et al. (2012); Goel (2012b)). This is an inflexible period in terms of Theorem 1. However, drivers are free to take a break at any other times.

Break constraints:

By leveraging the theorem, we can model the SH rules as follows:

$$x^B_j \leqslant x^B_i + \sum_{b\in\mathcal{B}} bx^b_{ij} + M(1 - x_{ij}), \qquad \text{for all } (i,j) \in A, \tag{13}$$

$$(x^s_j + s_j + s^2_{l_j}) - (x^s_i - s^1_{l_i}) \leqslant 300 + 150y^{15}_{ij} + 315y^{30}_{ij} + 330y^{45}_{ij} + (t_{max} - 300)y^{60}_{ij}, \qquad \text{for all } (i,j) \in H \tag{14}$$

$$x^B_j - x^B_i \geqslant \sum_{b\in\mathcal{B}} by^b_{ij} - M(1 - \sum_b y^b_{ij}), \qquad \text{for all } (i,j) \in H \tag{15}$$

$$\sum_b y^b_{ij} \leqslant 1 \qquad \text{for all } (i,j) \in H \tag{16}$$

$$x^b_{ij} \leqslant x_{ij} \qquad \text{for all } (i,j) \in A, b \in \mathcal{B}, \tag{17}$$

$$\sum_{b\in\mathcal{B}} x^b_{ij} \leqslant 1 \qquad \text{for all } (i,j) \in A. \tag{18}$$

Constraint 13 ensures that the total break time by the start of task j does not exceed the total break time by the start of the previous task i plus the total break taken between task i and j; that is,$\sum_{b\in\mathcal{B}} bx^b_{ij} + M(1 - x_{ij})$. Constraints 14 and 16 are the direct result of Theorem 1 and indicate the length of all intervals starting and ending with inflexible periods by variables y^b_{ij}. For every $(i,j) \in H$, the length of interval $[x^s_i - s^1_{l_i}, x^s_j + s_j + s^2_{l_j}]$ is either less than 302, or in $[302, 452]$, or in $[453, 617]$, or in $[618, 632]$, or in $[633, t_{max}]$, which by Theorem

1 requires 0, 15, 30, 45, 60 minutes break respectively. The minimum break time in intervals $[f_i, s_{i+1}]$ is enforced by Constraint 15. Constraint 17 ensures that x^b_{ij} takes value zero when task j does not succeed task i. Constraint 18 imposes that at most only one break variable for each pair of consecutive tasks can take a non-zero value. For example, if we have a 45-minute break, there are two possibilities without Constraint 18: $x^{15}_{ij} = 1, x^{30}_{ij} = 1$ or $x^{45}_{ij} = 1$. But the first possibility is not consistent with the definition of variables x^b_{ij}.

Remark 1. There are scenarios in which multiple short tasks are scheduled to be done consecutively at the same location. In these scenarios, scheduling a break between these short tasks may not be desirable due to service disruption. To prevent this, we replace constraints 13 with the following constraints:

$$x^B_j \leqslant x^B_i + \sum_{b\in\mathcal{B}} b x^b_{ij} + M(1 - x_{ij}), \qquad \text{for all } (i, j) \in A, \text{if } l_i \neq l_j \text{ or } t^i_i + s_i > t^l_j \tag{19}$$

$$x^B_j \leqslant x^B_i + M(1 - x_{ij}) \qquad \text{for all } (i, j) \in A, \text{if } l_i = l_j \text{ and } t^i_i + s_i \leqslant t^l_j, \tag{20}$$

If task i and j are at the same location and can be done back-to-back, constraint 20 ensures that scheduling a break between i and j is disallowed.

It may be of interest to some readers to know how we can deal with this problem when breaks can be scheduled at any time and at any location. Under the SH policy, we have the flexibility of any break location, but not the flexibility of any break time. For simplicity, assume that the preparation times are zero. Under "any time and any location" scenario, for each task the time window should be defined for the whole service time, and the service time should be contained in the time window. In addition to the task start time variable, we need to define a task end time variable. We also need to define additional variables for breaks happening within tasks and between tasks at the same location.

Remark 2. Goel (2012b) assumed that breaks can only take place at customer locations before the service starts. This restriction can be modelled by a slight modification to the model for the SH policy. Constraint 14 will change to:

$$(x^s_j + s_j + s^2_{l_j} + \sum_{(j,k)\in H} d_{jk} x_{jk}) - (x^s_i - s^1_{l_i}) \leqslant$$
$$300 + 150 y^{15}_{ij} + 315 y^{30}_{ij} + 330 y^{45}_{ij} + (t_{max} - 300) y^{60}_{ij}, \qquad \text{for all } (i, j) \in H \tag{21}$$

This constraint for each (i, j) corresponds to the interval

$$[x^s_i - s^1_{l_i}, x^s_j + s_j + s^2_{l_j} + \sum_{(j,k)\in H} d_{jk} x_{jk})]$$

which contains the inflexible interval $[x^s_i - s^1_{l_i}, x^s_i + s_i + s^2_{l_i}]$ at the start and the inflexible interval

$$[x^s_j - s^1_{l_j}, x^s_j + s_j + s^2_{l_j} + \sum_{(j,k)\in H} d_{jk} x_{jk})]$$

at the end.The latter interval contains service period at location j and the subsequent travel period. Since no break can take place after the service j or while travelling, Constraint 21 immediately follows from Theorem 1.

4.2 The SHD policy

Under the SHD policy, breaks can only occur at a depot. In a situation with only one depot, if there needs to be a break between two consecutive tasks, the driver should go to the depot, take the break, and continue the tour. The model for the SH policy needs two major changes in the precedence constraints and the break constraints.

Precedence constraints:

$$x_i^s + s_{ij} + d_{ij}(1 - \sum_b x_{ij}^b) + (d_{i0} + d_{0j}) \sum_b x_{ij}^b + \sum_{b \in \mathcal{B}} b x_{ij}^b \leqslant x_j^s + M(1 - x_{ij}) \qquad \text{for all } (i,j) \in A. \qquad (22)$$

If we have a break between task i and task j, the driver needs to drive from location l_i to depot, take break, and then drive to location l_j. The total driving time between task i and j is $(d_{i0} + d_{0j})$.

Break constraints:

In this setting, the inflexible intervals either commence at the start of a preparation or at the departure from the depot after taking a break. Moreover, they end either at the end of an after-task preparation or on arrival at the depot before the break.

$$x_j^B \leqslant x_i^B + \sum_{b \in \mathcal{B}} b x_{ij}^b + M(1 - x_{ij}), \qquad \text{for all } (i,j) \in A, \qquad (23)$$

$$(x_j^s + d_{j0}(\sum_{\substack{k:(j,k)\in A,\\ b\in\mathcal{B}}} x_{jk}^b) + s_j + s_{l_j}^2) - (x_i^s - d_{0i}(\sum_{\substack{i:(k,i)\in A,\\ b\in\mathcal{B}}} x_{ki}^b) - s_{l_i}^1) \leqslant$$

$$300 + 150 y_{ij}^{15} + 315 y_{ij}^{30} + 330 y_{ij}^{45} + (t_{max} - 300) y_{ij}^{60}, \qquad \text{for all } (i,j) \in H \qquad (24)$$

$$x_j^B - x_i^B \geqslant \sum_{b \in \mathcal{B}} b y_{ij}^b - M(1 - \sum_b y_{ij}^b), \qquad \text{for all } (i,j) \in H \qquad (25)$$

$$\sum_b y_{ij}^b \leqslant 1 \qquad \text{for all } (i,j) \in H \qquad (26)$$

$$x_{ij}^b \leqslant x_{ij} \qquad \text{for all } (i,j) \in A, b \in \mathcal{B}, \qquad (27)$$

$$\sum_{b \in \mathcal{B}} x_{ij}^b \leqslant 1 \qquad \text{for all } (i,j) \in A. \qquad (28)$$

Note that the inflexible intervals might have a different structure compared to the those of the SH policy. The inflexible interval for (i,j) has this structure:

$$\left[x_i^s - d_{0i}(\sum_{\substack{i:(k,i)\in A,\\ b\in\mathcal{B}}} x_{ki}^b) - s_{l_i}^1, x_j^s + d_{j0}(\sum_{\substack{k:(j,k)\in A,\\ b\in\mathcal{B}}} x_{jk}^b) + s_j + s_{l_j}^2 \right]$$

If there is no break between task i and its previous task, and there is no break between task j and its next task, then the interval has the same structure as the corresponding interval in the SH policy; that is,

$$\left[x_i^s - s_{l_i}^1, x_j^s + s_j + s_{l_j}^2 \right]$$

If there is a break between task i and its previous task, since that break should be taken at the depot, the driver cannot take another break after departure from the depot and before task i. Analogously, if there is no break between task j and its next task, the driver cannot take another break after departure from task j and before arrival at the depot.

4.3 Longer break blocks

According to the SH rules, the minimum rest time is 15 minutes. This is often too short for a proper rest. In this section, we consider increasing the minimum break time. According to the Australian regulations, from any period of continuous break time,only the largest multiple of 15 minutes less than or equal to the break time is counted as a break. For example, 25 minutes break is counted as 15 minutes. So, one sensible way to increase the minimum rest time is to increase the length of break blocks from 15 minutes to 30 minutes. 30-minute rests are common in the logistics industry and are reflected in rest break regulations for truck drivers worldwide. In the US (Federal Register (2020)), the minimum rest break time in 30 minutes, and European regulations (Europa 1 (2020); Europa 2 (2020)) require at least one 30-minute continuous rest break during working hours of truck drivers.

Consider SH-type rule (330,1,30). Based on the rule, we need to have at least one 30-minute break block in every 330 minutes. This rule dominates the three SH rules introduced before. In other words, if we have a tour satisfying this rule, the SH rules are also satisfied. Consider tour $(s_i, f_i)_{[}n]$. Assume the length of the tour is less than 13 hours or 780 minutes. We can prove the following theorem similar to the proof of Theorem 1.

Theorem 2. *There exists an SH-feasible schedule of breaks if and only if for all $i, j \in [n], i \leqslant j$, we have:*

1. *if $f_j - s_i \geqslant 271$, then $\sum_{l=i}^{j-1} b_i \geqslant 1$,*
2. *if $f_j - s_i \geqslant 601$, then $\sum_{l=i}^{j-1} b_i \geqslant 2$.*

Furthermore, this feasible schedule can be obtained by just scheduling b_i break block(s) within flexible period $[f_i, s_{i+1}]$ comprised of $b_i\delta$ break time and $s_{i+1} - b_i\delta - f_i$ combined preemptive work time and idle time for every $i \in [n-1]$.

Proof. Analogous to the proof of Theorem 1. □

Theorem 2 introduces necessary and sufficient conditions for the rule (330,1,30). To model this new rule, we need to modify set $\mathcal{B}$ to $\{30, 60\}$, and constraints 14, and 24.

Constraint 14 is replaced with

$$(x_j^s + s_j + s_{l_j}^2) - (x_i^s - s_{l_i}^1) \leqslant 270 + 330y_{ij}^{30} + (t_{max} - 270)y_{ij}^{60}, \qquad \text{for all } (i,j) \in H, \tag{29}$$

and constraint 24 with

$$(x_j^s + d_{j0}(\sum_{\substack{k:(j,k)\in A,\\ b\in\mathcal{B}}} x_{jk}^b) + s_j + s_{l_j}^2) - (x_i^s - d_{0i}(\sum_{\substack{i:(k,i)\in A,\\ b\in\mathcal{B}}} x_{ki}^b) - s_{l_i}^1) \leqslant \tag{30}$$

$$270 + 330y_{ij}^{30} + (t_{max} - 270)y_{ij}^{60} \qquad \text{for all } (i,j) \in H.$$

Corresponding to this rule, we define two new policies named Standard Hours with Longer break blocks (SHL policy) and Standard Hours with breaks at Depot with Longer breaks (SHDL policy).

5 Computational Study and Discussion

In our computational studies, we used Gurobi 8.1 Optimiser on a 64-bit Windows 10 Machine with 16 GB of RAM and Intel 4.8GHz i7-8665U processor. To compare policies, we generate 176 benchmark classes each comprised of 7 instances. Each class corresponds to an actual tour that is run daily by a major postal company in Australia (data is related to the tours in Sydney Metropolitan and its regional areas). In each class, the instances differ only in their time windows. For each tour, we have the tasks to be completed and the capacity of the truck assigned to it. For each task, we have its duration, the scheduled start time, and the location. For each pickup task, we know its associated delivery task.

We observe in our dataset that the deliveries are not tightly scheduled by the company. This is consistent with the current practice and industry norms for regular postal service. Since most items in our database are classified as non-express, we consider no specific delivery time for daily delivery tasks. Practically, the delivery times are usually bounded by the maximum length of the tours which is 12 hours. Therefore, we only need to generate time windows for pickup tasks.

In each class, we have 7 instances corresponding to $\mathcal{O} = \{5, 20, 35, 50, 65, 95, 125\}$. Let t_i^s denote the scheduled start time of pickup task i in instance $o \in \mathcal{O}$. The time window for task i, i.e. $\left[t_i^e, t_i^l\right]$, is equal to $[t_i^s - o, t_i^s + s_i + o]$. All preparation times are set to zero. Service duration ranges from 5 minutes to 55 minutes with an average of approximately 12 minutes. Distance matrix (in minutes) is pulled from google maps database. We use peak travel times on Tuesday, 24 September 2019, at 4:30PM as the tours were run on that date.

Locations of all tasks are depicted in Figure 1. The size of each circle corresponds to the number of daily visits to that location. The figure contains 240 locations across Sydney Metropolitan region. The top two pickup locations (the two largest circles in the figure) comprise 37% of all visits, and the top two delivery locations (the two largest circles in the figure) comprise 47% of all visits. These locations host the two major sortation facilities for mail and parcels.

Most of the tours require trucks with capacities of 37 cubic meters (62 percent of the tours) and 58 cubic meters (19 percent of the tours). The capacity requirement of a tour is an input to the model which can be determined based on the road access restrictions and customer requirements.

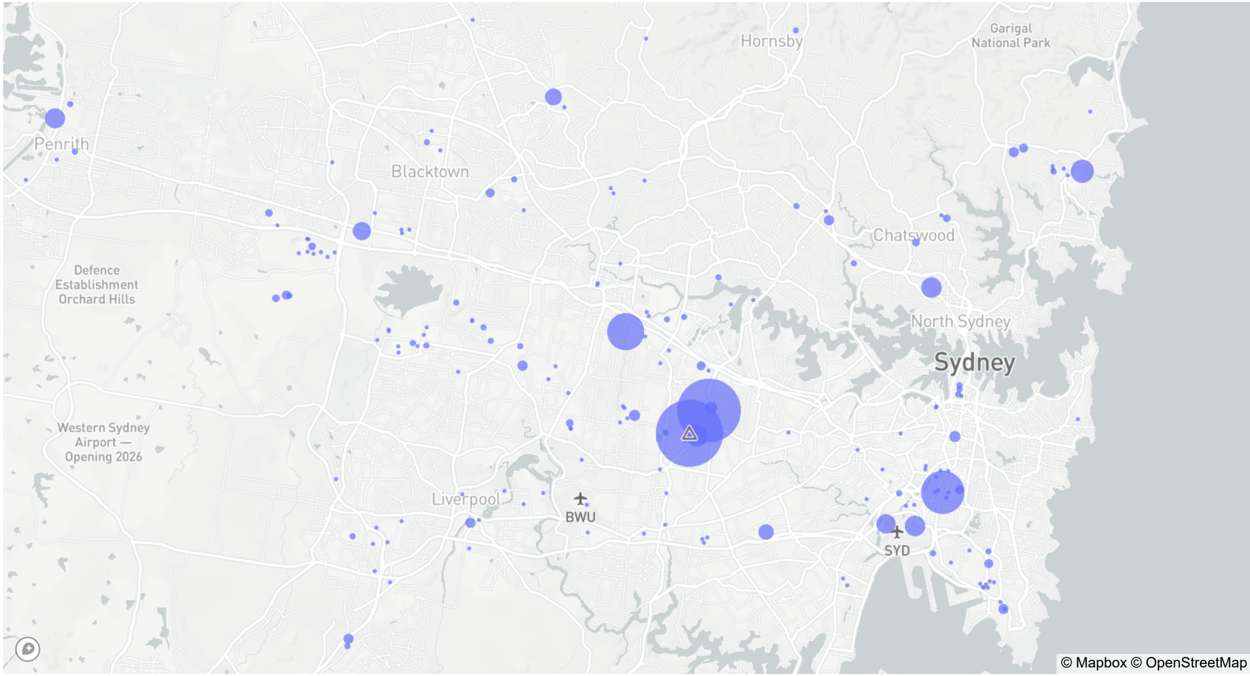
In Table 3, we presented a typical tour as planned by the postal carrier. A 30-minute break is scheduled at the depot. The tour has 12 tasks with the first task starting at 12:10 at the depot and the last task ending at 21:05 at the depot. This tour has to be executed from Monday to Friday every day.

Table 3: A typical tour time and duration (locations anonymised)

Transport Tour: 341
Monday to Friday 12:10-21:05

Arrive	**Location**	**Instructions**	**Depart**
12:10	Depot	Prepare a small truck	12:25
12:50	Customer 1	Collect All Available for Hub 1	13:05
14:10	Hub 1	Deliver All Available ex - Customer 1	14:20
15:05	Facility 1	Collect All Available for Hub 2	15:25
15:50	Hub 2	Deliver All Available ex - Facility 1	16:00
16:05	Depot	Rest Break	16:35
17:40	Customer 2	Collect All Available for Customer 3	18:35
18:25	Customer 3	Deliver All Available ex - Customer 2	
		Collect All Available for Hub 2	18:35
19:20	Facility 2	Collect All Available for Hub 1, Hub 2	19:40
20:25	Hub 2	Deliver All Available ex - Facility 2, Customer 3	20:35
20:45	Hub 1	Deliver All Available ex - Facility 2	20:55
21:00	Depot	Return and Refuel Vehicle	21:05

Figure 1: Customer locations and their frequency of visits in Sydney



5.1 Analysis of tour duration

Tables 4 and 5 compare the optimal tour times of the SH policy and the SHD policy with the no break scheme. The columns, from left to right, show instance classes $\mathcal{O}$, mean difference, standard deviation, minimum difference, first quantile, second quantile, third quantile, and maximum difference in minute (m) and percentage (%). For each class $o \in \mathcal{O}$, we only consider tours that have optimal solutions under all three policies. All numbers in the tables are rounded down.

The second column shows the mean duration difference of optimal tours when compared to the no break scheme. Let us refer to class $o \in \mathcal{O}$ as offset-o class. For offset 5 in Table 4, the mean difference is 3.2 which

Table 4: Difference in tour duration between the SH policy and the no break scheme

offset	mean		std		min		FQ		SQ		TQ		max	
	min	%	min	%	min	%	min	%	min	%	min	%	min	%
5	3.2	0.6	7.5	1.5	0	0	0	0	0	0	0	0	30	7
20	7.2	1.5	10.8	2.2	0	0	0	0	0	0	12	3	30	7
35	11.6	2.5	12.0	2.5	0	0	0	0	10	2	19	4	30	7
50	14.5	3.2	11.7	2.4	0	0	1	0	15	4	30	6	30	7
65	17.2	3.8	11.3	2.3	0	0	14	3	15	4	30	6	30	7
95	18.3	4.2	10.3	2.0	0	0	15	4	15	4	30	6	30	7
125	18.6	4.2	10.2	2.0	0	0	15	4	15	4	30	6	30	7

means that the average duration of optimal tours under the SH policy is 3.2 minutes longer than the average tour duration under the no break scheme. As the offset gets bigger, the difference converges to a value equal to the average break time. This is because when the tasks can be scheduled freely, the problems of task sequencing and break scheduling become two independent problems. In other words, we can first find an optimal sequence of tasks and then schedule the breaks. When this happens, the difference between optimal scheduled tour under the SH policy and optimal tour under the no break scheme for every instance will be equal to the total break time.

There are two factors at play here. The first factor is the time window and the second is the break policy. Bigger time windows allow for more flexible sequencing of tasks in each tour. Of course, requiring a break limits this positive effect of wider time windows on the tour length. As time windows get larger, the mean difference, unsurprisingly, converges to the minimum required break. An evidence of this would be the quantile statistics with the majority being at 0, 15, and 30 minutes.

Table 4 also shows the average difference, in percentage, between the SH policy and the no break scheme. As offsets get bigger the average difference increases slowly from 0.6% at offset 5 to 4.2% at offset 125. To get a better sense of these numbers, let us work out another way of estimating the average difference. For each optimal tour under the no break scheme, we calculate the minimum break time by applying Theorem 1 on the optimal tour duration. On average, the obtained minimum break is equal to 4.85% of the optimal tour duration. That is, if optimal tours have no idle times that can be used for rest breaks, we expect that the optimal tour lengths increase at least by 4.85% under the SH policy. However, the average idle times of optimal tours under the no break scheme is 5%. Therefore, the increase could also be less than 4.85%. In Table 4, we observe that the mean differences, especially for bigger offsets, match 4.85%. In general, as offsets get bigger, for each instance, the idle times approach zero for all policies and the difference between the SH policy and the no break scheme approach the total break time.

In Table 4, the minimum difference in tour duration for all offsets in $\mathcal{O}$ is 0. This means that there are instances that the optimal tour length under both policies are equal. The main characteristic of these instances is that there exists at least one optimal tour under the no-break scheme that the required break times under the SH policy can be scheduled during the idle times in the tour. Therefore, for those instance, there exists an optimal tour under the no-break scheme that is also an optimal tour under the SH policy. The idle times can exist in an optimal tour because of the time windows and because it is not always possible to schedule back-to-back tasks with no slack time in between.

Under the SHD policy (Tables 5), the mean difference compared to the no break scheme is much larger than what is reported in Table 4. This is pretty much expected since the drivers under the SHD policy have to travel the extra miles to take rest at the depot. Perhaps the more interesting difference compared to the SH policy is the standard deviation being almost twice as much for the SHD policy when compared to the SH policy. The maximum difference also is much higher for the SHD policy compared to the SH policy.

To facilitate a better comparison between the two schemes, we provide a direct comparison between the SHD policy and the SH policy in Table 6. Quantiles suggest that, in a majority of instances, both policies have rather identical performance in terms of tour duration. This can be explained by the high frequency of visits to two of strategically-located sortation facilities.One of these facilities is very close to the depot and the other co-locates with the depot which makes it very convenient as they are visited frequently by many tours. The depot is marked by a triangle in Figure 1. This way, taking a break at the depot becomes much less costly as the drivers do not require travelling significant extra miles just for the purpose of a break. This justifies a small different of only 1-1.5 percent between the average tour length of the SHD and that of the SH policies. The gains may seem slim in the short term, but the overall benefits of the SH policy become more pronounced in the long term.

The distribution of benefits across all tours is not uniform and there are extreme cases in which the extra miles are significant (see the last two columns of Table 6). In those cases, the break needs to be scheduled at different locations, not just at the depot. There are, therefore, tours that may benefit from scheduling the rest breaks in locations other than the depot. The benefits in these situations can be as large as 30% reduction in tour length which is quite significant.

Under the SHD policy, tours have to travel extra miles on average compared to the SH policy. Extra miles are not generally desirable as they prompt further uncertainties in travel times. This is a bigger issue in larger cities like Sydney with very congested roads during peak ours. For parcel collection services, this is even more pronounced when a large share of tasks is scheduled during the peak traffic hours (7:00 to 10:00 in the morning and 4:00 to 7:00 in the afternoon, Liao et al. (2020)). In our dataset, around 21 % of all tasks are scheduled in the peak hours. Any reduction in the frequency and length of the tours, especially in the most congested areas, could reduce uncertainties in travel times for planning purposes, and contribute to improved city congestion as a whole.

Table 5: Difference in tour duration between the SHD policy and the no break scheme

offset	mean		std		min		FQ		SQ		TQ		max	
	min	%	min	%	min	%	min	%	min	%	min	%	min	%
5	9.0	1.8	24.8	4.9	0	0	0	0	0	0	7	1	172	35
20	11.7	2.4	19.8	4.1	0	0	0	0	0	0	16	4	137	30
35	16.0	3.5	19.2	4.1	0	0	0	0	13	3	30	6	136	30
50	19.0	4.2	18.7	4.0	0	0	6	2	15	4	30	6	136	30
65	22.6	5.1	19.2	4.2	0	0	15	4	15	5	30	6	136	30
95	24.4	5.6	21.0	4.8	0	0	15	4	15	5	30	6	136	30
125	23.0	5.3	18.9	4.3	0	0	15	4	15	5	30	6	136	30

In Table 5, the minimum difference in tour duration for all offsets is 0 (similar to what we observed in Table 4). This means that there are instances that the optimal tour length under both policies are equal. The main characteristic of those instances is that there exists at least one optimal tour under the no-break policy that the required break times plus the travel times to and from the depot can be scheduled during idle times in the tour. Therefore, for those instances, there exists an optimal tour under no-break scheme that is also an optimal tour under SHD policy. Since the depot co-locates with one of the most frequently visited hubs by many tours, there are more than a few instances with the mentioned characteristics.

Table 6: Difference in tour duration between the SHD policy and the SH policy

offset	mean		std		min		FQ		SQ		TQ		max	
	min	%	min	%	min	%	min	%	min	%	min	%	min	%
5	5.9	1.1	22.1	4.4	0	0	0	0	0	0	0	0	148	30
20	4.5	1.0	14.6	3.1	0	0	0	0	0	0	0	0	111	23
35	4.4	1.0	13.0	2.9	0	0	0	0	0	0	1	0	107	24
50	4.5	1.0	13.6	3.1	0	0	0	0	0	0	1	0	106	24
65	5.4	1.3	15.9	3.7	0	0	0	0	0	0	1	0	106	24
95	6.2	1.5	18.3	4.3	0	0	0	0	0	0	3	1	112	26
125	4.4	1.0	14.6	3.6	0	0	0	0	0	0	0	0	103	26

In our analysis, we compared two representative policies. One policy with no restriction on rest break location, and the other policy with a restriction to only take a break at the depot. A policy with multiple designated rest break locations cannot be more expensive than the SHD policy (in terms of tour duration), and cannot be less expensive than the SH policy either. We thus studied the two extreme situations as representative scenarios. The developed framework for modelling these policies is highly flexible and can handle many rest break scenarios. We considered three more scenarios in Remark 2, and Section 4.3.

5.2 Analysis of travel time

Heavy-duty trucks are one of the largest sources of emissions generation and energy use. Globally, greenhouse emissions from heavy-duty vehicles are expected to surpass emissions from passenger vehicles by 2030 Federal Register (2015). This suggests that from both environmental and efficiency perspectives, it is important to understand the impact of scheduling/planning policies and practices on fuel consumption and greenhouse emissions (Chen et al. (2020); Richardson (2005)).

To provide a comparison between the SH policy and the SHD policy in terms of fuel consumption and emissions, we compare optimal total travel time of the optimised tours. To obtain optimal total travel times, we use a hierarchical optimisation approach. We first optimise based on the tour length, and then optimise based on the total travel time while fixing the tour length at the optimal value.

Table 7: Difference in minimum travel time between the SHD policy and the SH policy

offset	mean	min	max	posdiff(%)	negdiff(%)
5	11.8	0	200	32	0.0
20	9.7	0	181	30	0.0
35	6.1	-22	181	29	1.4
50	6.9	-1	181	24	0.7
65	6.3	-4	181	22	1.5
95	5.8	-15	181	23	0.8
125	3.0	-1	103	18	0.9

Table 8: Average share of the minimum travel time in the optimal tour duration

offset	meanSHD(%)	meanSH(%)	meandiff
5	54.5	53.0	1.5
20	57.4	56.0	1.4
35	58.0	57.2	0.8
50	58.9	58.0	0.9
65	59.0	58.3	0.7
95	58.9	58.4	0.5
125	59.2	59.0	0.2

The aggregate results are provided in Table 7. Columns mean, min, and max show the average, minimum, and maximum difference (measured in minutes) between the minimum total travel time under the SH policy from those of the SHD policy across tours. Columns posdiff and negdiff show the percentage of instances in which the SHD policy and the SH policy, respectively, have bigger minimum travel times. As expected, the SHD policy has larger minimum total travel time on average. However, the difference gets smaller as the offset gets bigger or the time windows get wider. This is because of the extra flexibility afforded by the wider time windows in scheduling and sequencing tasks that almost dominates the inflexibility introduced by taking the break at the depot. In all instance groups corresponding to offsets, there are instances in which either the SHD policy or the SH policy has greater minimum total travel time. On average, across offsets, in 0.75% of instances the SH policy requires more travel time than the SHD policy. This figure is 25% for the SHD policy, meaning that the SHD policy, on average, requires more travel time in 25% of instances.

Table 8 shows the average share of the minimum travel time in the optimal tour duration for each offset group. Columns meanSHD, meanSH, and meandiff show the average share under the SHD policy, the SH policy, and the mean difference of the SHD policy from the SH policy. On average, the percentage of minimum travel time in an optimal tour is 58% under the SHD policy and 57% under the SH policy. So, on average, SHD policy compliant tours require slightly longer travel times. This is consistent across all instance groups, but not across all instances - meaning that there are some instances in which the SH-optimal tours have longer travel times. The difference between the two polices fades away as offsets get bigger.

It is somehow counter intuitive that there are instances, albeit rare, in which the SH-optimal tours have larger total travel times than those of the SHD-optimal tours. One analogy is that under the SHD policy there is at least one more location to visit for taking a break. This would have been true if the optimal sequence under both policies were the same. Before elaborating further, we define idle time of a tour as the total time of the tour that is neither a travel time, nor a service time. By this definition, the idle time of a tour includes the break time. Therefore, the duration of a tour can be partitioned into three time portions: total travel time, total idle time (including the break times), and total service time. The total service time for each tour is fixed under both polices. The total travel time and total idle time can change. When the total travel time for the SHD policy is less than the total travel time for the SH policy, the total idle time under the SH policy would compensate for or exceed the difference in total travel times. Otherwise the optimal tour duration for the SH policy would be bigger than that of the SHD policy. In most instances, the mandate to take a break at the depot under the SHD policy forces the tours to either travel extra miles to/from the depot for taking the break, or to take the break at times which might lead to extra waiting or idle times. Furthermore, the extra travel times can lead to extra mandatory break times.

5.3 The impact of longer break blocks

In this section, we analyse the impact of longer break blocks for each policy. We compare the SHL policy with the SH policy, the SHDL policy with the SHD policy, and the SHDL policy with the SHL policy.

Table 9: Difference in tour duration between the SHL policy and the SH policy

offset	mean		std		min		FQ		SQ		TQ		max	
	min	%	min	%	min	%	min	%	min	%	min	%	min	%
5	3.1	0.6	8.2	1.5	0	0	0	0	0	0	0	0	30	6
20	4.6	1.0	8.5	1.7	0	0	0	0	0	0	6	1	30	6
35	6.0	1.3	8.7	1.8	0	0	0	0	0	0	14	3	30	6
50	6.8	1.6	8.3	1.9	0	0	0	0	0	0	15	4	30	5
65	7.0	1.7	7.9	1.9	0	0	0	0	2	0	15	4	30	5
95	8.6	2.2	8.8	2.3	0	0	0	0	12	3	15	4	30	10
125	8.6	2.2	8.7	2.4	0	0	0	0	15	3	15	4	30	10

Table 9 presents the optimal tour duration comparison for the SHL and SH policies, and Table 10 compares the SHDL policy with and the SHD. We observe that the difference between the two policies is larger, on average, for the larger offsets. However, this trend does not hold across all instances. We know that optimal tour duration under the SHL policy should be at least as much as that of the SH policy, and under thr SHDL policy should be at least as much as that of the SHD policy. For instances that require a 15-minute break in the optimal tour under the SH or SHD policies, they require at least 30 minutes break under the SHL or SHDL policies, respectively. For instances that require a 30-minute break in the optimal tour under the SH or

SHD policies, they require at least 30 minutes break under the SHL or SHDL policies, and so on and so forth. As the offset gets larger, under the SH or SHD policies there are more opportunities for tour length reduction for many reasons. Firstly, the mandatory total break is often less. Secondly, even when the mandatory total break is the same, there is more flexibility under the SH or SHD policies in comparison with the SHL and SHDL policies, because the break blocks are smaller.

For offsets above certain threshold at which the tasks can be scheduled freely, the difference between the SHL and SH policies is fixed as they no longer impact the optimal sequence. In other words, finding the optimal sequence becomes independent of the scheduling problem which includes the scheduling of breaks. The same conclusion holds for the SHDL and SHD policies but with this twist that when the time windows are not binding and tasks can move freely, the mandate to take a break at the depot might still affect the optimal sequence. In this case, it is not obvious whether or not the break scheduling and sequencing problems are independent of each other.

Table 10: Difference in tour duration between the SHDL policy and the SHD policy

offset	mean		std		min		FQ		SQ		TQ		max	
	min	%	min	%	min	%	min	%	min	%	min	%	min	%
5	8.0	1.5	14.2	2.7	0	0	0	0	0	0	8	2	60	11
20	11.1	2.2	14.5	2.7	0	0	0	0	2	0	20	4	89	15
35	16.0	3.3	27.5	5.3	0	0	0	0	13	3	22	4	257	50
50	15.4	3.3	25.9	5.2	0	0	0	0	15	3	15	4	260	51
65	17.2	3.9	29.9	6.7	0	0	0	0	15	4	17	5	245	56
95	20.1	4.6	30.3	6.6	0	0	0	0	15	4	30	5	229	47
125	21.0	4.8	34.2	7.6	0	0	0	0	15	4	30	5	229	45

Table 11: Difference in tour duration between the SHDL policy and the SHL policy

offset	mean		std		min		FQ		SQ		TQ		max	
	min	%	min	%	min	%	min	%	min	%	min	%	min	%
5	6.0	1.2	11.9	2.4	0	0	0	0	0	0	5	1	47	10
20	10.6	2.1	23.2	4.6	0	0	0	0	0	0	13	3	185	36
35	13.8	2.8	32.4	6.6	0	0	0	0	0	0	13	3	257	50
50	12.6	2.6	30.0	6.1	0	0	0	0	0	0	13	3	260	51
65	15.4	3.4	35.4	8.0	0	0	0	0	0	0	14	3	245	57
95	17.4	3.7	36.6	8.0	0	0	0	0	0	0	22	4	229	47
125	16.5	3.5	39.5	8.7	0	0	0	0	0	0	9	2	229	45

Table 11 shows the difference between the SHDL and SHL policies. The difference in this case is larger than the difference between the SHD and SH polices. We conclude that the increase in the minimum break duration intensifies the negative impact of enforcing break(s) at the depot on the optimal tour lengths. With shorter minimum break times, it is easier to accommodate the depot break(s) in the tours while still meeting the time windows without significantly changing the optimal sequence with no break.

Apart from the HOS regulations, there are other restrictions that may affect the scheduling of rest breaks. In Australia, there are four sources of rules and regulations: legislation, awards, registered/enterprise agreements, and employment contracts. These rules and regulations are issued and implemented at different levels in terms of coverage and application. At the broadest level, the minimum work entitlements (including the

HOS regulations) are legislated by the federal and/or state governments. These are commonly referred to as legislation. Awards are legal documents that set the minimum standards and pays for specific industries or occupations. Registered/enterprise agreements set out the minimum employment conditions for a specific organisation or a group of organisations (i.e., agreement between the Fair Work Commission and an individual organisation or a specific industry). Finally, the individual level of coverage is the employment contracts through which certain work conditions may be applied to an individual or group of individuals. Our focus in this paper was merely on addressing the HOS regulations enforced at the federal/state level since most enterprise agreements are often defined under the umbrella of those broader federal regulations. However, studying the interactions between the federal/state regulatory mandates and industry/corporate/individual agreements can be intricate at times when multiple objectives need to be satisfied in a single scheduling/planning problem. Investigation of these interactions, perhaps on a case-by-case or industry-by-industry basis, is an important and interesting direction for future research in this area.

6 Conclusion

This paper presented a MIP-based framework for modelling a class of rest break policies for truck drivers. We model and compare two representative policies from this class named SH and SHD policies. In the SH policy, all breaks are scheduled according to the standard hours regulations. In the SHD policy, the breaks are still scheduled according to the standard hours regulations with an extra constraint that enforces the breaks to be only taken at the depot. Logically, the SHD policy should be more expensive in terms of tour length due to more restrictive rest break location. However, we observe in a real case study from postal services in Australia that the magnitude of benefits that can be obtained from the SH policy is not as large as one would expect. Our computational analysis suggests that the SHD policy, on average, is between 1 to 1.5 percent more expensive than the SH policy. The reason is that most of the tours in the concerned service area (Sydney metropolitan) occur in the vicinity of the depot (where the rest breaks occur in the SHD policy); hence, the extra miles that need to be travelled to return to the depot to take a break may not be as significant. This finding reinforces the important role that facility location (depot location in this case) plays in policy impact analysis. This level of analysis is crucial when making supply chain network design decisions as the outcomes not only impacts the internal scheduling decisions, but they also contribute to the pick-time traffic congestion in a broader perspective.

The framework is developed taking into consideration the necessary and sufficient conditions pertaining to the application of the SH rules for short daily work hours. These conditions allows us to not explicitly schedule breaks to find the optimal tour length. They also enable us to model rest break requirements without losing the tractability using the existing MIP solvers. However, with these conditions we imply that we are not seeking an exact solution to the break schedule problem. This makes sense for the purpose of our analysis because having an exact schedule of breaks is not a requirement for finding an optimal tour.

The proposed framework was intended to be simple to use by business analysts with basic knowledge of MIP modelling. The rapidly changing business environment of today necessitates frequent changes/updates in analytical models and/or the development of new models. MIP models are versatile and can handle new requirements relatively easily. In many situations, the powerful MIP solvers, commercial or open source, can solve standard industrial-sized problems, rendering the need for the customised advanced algorithms defunct. Developing advanced solution algorithms is time-consuming, demands high technical knowledge (hence not practical in real world), and requires ongoing maintenance; thus, they should be avoided if unnecessary.

There are multiple directions through which this research can be extended. One direction is to extend our framework and analysis to compare policies using other measures such as the number of vehicles in a multi-vehicle scenario (note that our intention in this paper was to compare the rest break policies for a single vehicle/driver to avoid service interruption/inconsistency due to changes in the set of tasks in each tour).

Another interesting direction for future research is to develop algorithms to find an exact schedule of breaks in an optimal tour under the SH policy with unrestricted rest break location. Given that our approach gives the duration of the rest break between tasks, finding an exact schedule of breaks should be plausible. Extension of the developed models to accommodate longer work hours which is essential for intercity planning is another future research avenue.

Acknowledgments

This research was funded by iMOVE CRC, a research funding initiative under the Australian Government's Cooperative Research Centres (CRC) program (Grant ID: 199789). The authors are thankful to the University of Sydney Business School and the industry partner for partial funding of this project as part of the iMOVE CRC funding scheme. The authors are also grateful to Thomas Kalinowski for his insightful comments to develop and improve the theorems.

References

Archetti, C. and M. Savelsbergh (2009, November). The Trip Scheduling Problem. *Transportation Science 43*(4), 417–431.

Briest, P., J. Dragendorf, T. Ecker, D. Mohr, and F. Neuhaus (2019). *The Endgame for Postal Networks: How to Win in the Age of e-Commerce.* McKinsey & Company.

Ceselli, A., G. Righini, and M. Salani (2009, February). A Column Generation Algorithm for a Rich Vehicle-Routing Problem. *Transportation Science 43*(1), 56–69.

Chen, F., Z. Yin, Y. Ye, and D. Sun (2020, October). Taxi hailing choice behavior and economic benefit analysis of emission reduction based on multi-mode travel big data. *Transport Policy 97*, 73–84.

Dong, B., M. Christiansen, K. Fagerholt, and S. Chandra (2020, November). Design of a sustainable maritime multi-modal distribution network – Case study from automotive logistics. *Transportation Research Part E: Logistics and Transportation Review 143*, 102086. 00003.

Dumas, Y., J. Desrosiers, and F. Soumis (1991, September). The pickup and delivery problem with time windows. *European Journal of Operational Research 54*(1), 7–22.

Europa 1 (accessed on July 30,2020). *Human resources:Transport sector workers:Road transportation workers.* `https://europa.eu/youreurope/business/human-resources/transport-sector-workers/road-transportation-workers/index_en.htm`.

Europa 2 (accessed on July 30,2020). *Regulation (EC) No 561/2006 of the European Parliament and of the Council of 15 March 2006.* `https://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=CELEX:32006R0561:EN:HTML#d1e817-1-1`.

Fahimnia, B., M. Bell GH, D. Hensher A, and J. Sarkis (2015). *Green Logistics & Transportation: A Sustainable Supply Chain Perspective.* Springer.

Federal Register (2015, July). *Greenhouse Gas Emissions and Fuel Efficiency Standards for Medium- and Heavy-Duty Engines and Vehicles— Phase 2.* `https://www.govinfo.gov/content/pkg/FR-2015-07-13/pdf/2015-15500.pdf`.

Federal Register (2020, January). *Hours of Service of Drivers.* https://www.federalregister.gov/documents/2020/06/01/2020-11469/hours-of-service-of-drivers.

Goel, A. (2009, February). Vehicle Scheduling and Routing with Drivers' Working Hours. *Transportation Science 43*(1), 17–26.

Goel, A. (2010, November). Truck Driver Scheduling in the European Union. *Transportation Science 44*(4), 429–441.

Goel, A. (2012a, December). The minimum duration truck driver scheduling problem. *EURO Journal on Transportation and Logistics 1*(4), 285–306.

Goel, A. (2012b, August). A mixed integer programming formulation and effective cuts for minimising schedule durations of Australian truck drivers. *Journal of Scheduling*.

Goel, A. (2014, May). Hours of service regulations in the United States and the 2013 rule change. *Transport Policy 33*, 48–55.

Goel, A., C. Archetti, and M. Savelsbergh (2012, May). Truck driver scheduling in Australia. *Computers & Operations Research 39*(5), 1122–1132.

Goel, A. and S. Irnich (2017, May). An Exact Method for Vehicle Routing and Truck Driver Scheduling Problems. *Transportation Science 51*(2), 737–754.

Goel, A. and L.-M. Rousseau (2012, December). Truck driver scheduling in Canada. *Journal of Scheduling 15*(6), 783–799.

Goel, A. and T. Vidal (2014, August). Hours of Service Regulations in Road Freight Transport: An Optimization-Based International Assessment. *Transportation Science 48*(3), 391–412.

Justice Laws (accessed on July 30,2020). *Commercial Vehicle Drivers Hours of Service Regulations.* https://laws-lois.justice.gc.ca/eng/regulations/SOR-2005-313/.

Koç, Ç., T. Bektaş, O. Jabali, and G. Laporte (2016, November). A comparison of three idling options in long-haul truck scheduling. *Transportation Research Part B: Methodological 93*, 631–647.

Koç, Ç., O. Jabali, and G. Laporte (2017, April). Long-haul vehicle routing and scheduling with idling options. *Journal of the Operational Research Society*.

Kok, A. L., C. M. Meyer, H. Kopfer, and J. M. J. Schutten (2010, September). A Dynamic Programming Heuristic for the Vehicle Routing Problem with Time Windows and European Community Social Legislation. *Transportation Science 44*(4), 442–454.

Li, X., Y. Ding, K. Pan, D. Jiang, and Y. Aneja (2020, August). Single-path service network design problem with resource constraints. *Transportation Research Part E: Logistics and Transportation Review 140*, 101945. 00003.

Liao, Y., J. Gil, R. H. M. Pereira, S. Yeh, and V. Verendel (2020, March). Disparities in travel times between car and transit: Spatiotemporal patterns in cities. *Scientific Reports 10*(1), 4056.

NHVR (accessed on July 30, 2020). *Work and Rest Requirements.* https://www.nhvr.gov.au/safety-accreditation-compliance/fatigue-management/work-and-rest-requirements.

Prescott-Gagnon, E., G. Desaulniers, M. Drexl, and L.-M. Rousseau (2010, November). European Driver Rules in Vehicle Routing with Time Windows. *Transportation Science 44*(4), 455–473.

PwC (2016). *Shifting Patterns: The Future of the Logistics Industry.* PwC Network.

Rancourt, M.-E., J.-F. Cordeau, and G. Laporte (2013, February). Long-Haul Vehicle Routing and Scheduling with Working Hour Rules. *Transportation Science 47*(1), 81–107.

Richardson, B. C. (2005, March). Sustainable transport: Analysis frameworks. *Journal of Transport Geography 13*(1), 29–39.

Ropke, S. and J.-F. Cordeau (2009, August). Branch and Cut and Price for the Pickup and Delivery Problem with Time Windows. *Transportation Science 43*(3), 267–286.

Sartori, C. S., P. Smet, and G. Vanden Berghe (2021, June). Scheduling truck drivers with interdependent routes under European Union regulations. *European Journal of Operational Research*, S0377221721005348.

Schiffer, M., M. Schneider, G. Walther, and G. Laporte (2019, March). Vehicle Routing and Location Routing with Intermediate Stops: A Review. *Transportation Science 53*(2), 319–343.

Tilk, C. and A. Goel (2020, May). Bidirectional labeling for solving vehicle routing and truck driver scheduling problems. *European Journal of Operational Research 283*(1), 108–124.

Xu, H., Z.-L. Chen, S. Rajagopal, and S. Arunapuram (2003, August). Solving a Practical Pickup and Delivery Problem. *Transportation Science 37*(3), 347–364.

Zhen, L., C. Ma, K. Wang, L. Xiao, and W. Zhang (2020, March). Multi-depot multi-trip vehicle routing problem with time windows and release dates. *Transportation Research Part E: Logistics and Transportation Review 135*, 101866. 00012.

Appendix A: Proof of Theorem 1

Proof of Theorem 1

The necessity of conditions immediately follows from Lemma 1.

We prove the sufficiency by showing we can find a feasible solution by just shifting break blocks around inside their associated flexible interval . Consider intervals of the form $[t, t+a_k-1]$ where $a_1 = 330, a_2 = 480, a_3 = 660$ corresponding to three break rules $(330, 1, 15), (480, 2, 15)$, and $(660, 4, 15)$. Assume interval $[t, t+a_k-1]$ where $k \in [3]$ is an infeasible interval with the earliest finish time. If there are multiple intervals with earliest finish time, it is the smallest one. We consider all cases for $[t, t+a_k-1]$ and show that they all can be made feasible without affecting the feasibility of the intervals with earlier finish time or with the same finish time but smaller length. Both ends of the interval can be either within a flexible interval or inflexible interval . If one end is in flexible interval , it can either be within a break or outside of a break in the current schedule. So, there are 9 possibilities for the ends of the interval (i.e., 3×3) and there are 3 rules, which leads to 27 cases. To make the exposition easier, we denote a case by $xy-k$. x denotes where the start of the interval is located in time. If the start is in an inflexible interval , $x = U$, for a flexible interval and break $x = B$, and for a flexible interval and non-break $x = N$. Parameter k indicates the type of the rule. For example, $UB-1$ denotes the case that interval corresponds to rule 1, the start of the interval, i.e. t, is in an inflexible interval , and the end of the interval,i.e. $t+a_1-1$, is in a break. Without loss of generality, we assume the infinite intervals $(-\infty, s_1)$ and (f_n, ∞) are all break intervals.

UU-1, UN-1, UB-1, NU-1, NN-1, NB-1: If $[t, t+329]$ is infeasible, $[t-1, t+328]$ is also infeasible and this is a contradiction to interval $[t, t+329]$ being the infeasible interval with the earliest finish time.

BU-1: There should be a break ending at $t+13$ otherwise interval $[t-1, t+328]$ is infeasible which is a contradiction. We consider two cases for the break block:

Case 1: it can move one minute to the right.

In this case, we can make the interval feasible by moving the break block one minute to the right because it is not blocked by an inflexible interval but it might make another interval finishing earlier than $t+329$ infeasible. We just need to consider the intervals ending at $t+13$ because all the other relevant intervals are not affected adversely by moving break block $[t-1, t+13]$ one to the right. There are three possible cases. Either interval $[t-316, t+13]$ of length 330 or interval $[t-466, t+13]$ of length 480 or interval $[t-646, t+13]$ of length 660 becomes infeasible.

Case 1-1: Interval $[t-316, t+13]$ becomes infeasible

In this case, interval $[t-316, t+328]$ is longer than 480, so by the assumption that interval $[t, t+329]$ is the earliest finishing infeasible interval and there is no break block in $[t, t+329]$, there should be at least two break blocks inside $[t-316, t+13]$. This is a contradiction as moving break $[t-1, t+13]$ on minute to the right, still leaves one full break block inside interval $[t-316, t+13]$.

Case 1-2: $[t-466, t+13]$ becomes infeasible

In this case, the interval $[t-466, t+328]$ is longer than 660. So there should be at least 4 break blocks inside $[t-466, t+13]$. Analogously to the previous case, this is a contradiction.

Case 1-3: $[t-646, t+13]$ becomes infeasible

In this case, since the length of the tour is less than or equal to 780, the start of the tour cannot be earlier than $t+329-780 = t-449$. So there are at least 4 break blocks in $[t-645, t-451]$. This is a contradiction to shifting break block $[t-1, t+13]$ one to the right making interval $[t-646, t+13]$ infeasible.

Case 2: It cannot move one to the right.

It follows that there is an inflexible interval starting at $t+14$. Let t' be the end of the rightmost inflexible interval overlapping $t+329$. Let $|[a, b]|$ denote the the length of interval $[a, b]$, that is, $b-a+1$. We have $|[t+14, t']| \geqslant 301$. Therefore by the first condition, there should be at least one break block in $[t+14, t']$. It implies that there should be a break block inside $[t, t+329]$. This is a contradiction.

BN-1: The break block overlapping t should end at $t+13$ otherwise interval $[t-1, t+328]$ is infeasible which is a contradiction. We consider two cases.
Case 1: We can move the break block one to the right.
We need to show that moving the break block one to the right will not make any interval finishing earlier that=n $t+329$ infeasible. Any earlier finishing interval which contain interval $[t-1, t+14]$ will not be affected by this move. Therefore, we just need to consider intervals ending at $t+13$. We consider 330, 480, and 660 minute intervals finishing at $t+13$.
Case 1-1: interval $[t-316, t+13]$
Interval $[t-316, t+328]$ is longer than 480 and by the feasibility assumption of earlier starting intervals, it should contain at least two break blocks. Since there is no break block in $[t+14, t+329]$, the two break blocks should be in $[t-316, t+13]$ and moving the rightmost break block one to the right cannot make interval $[t-316, t+13]$ infeasible. This is a contradiction.
Case 1-2: interval $[t-466, t+13]$
Interval $[t-466, t+328]$ is longer than 660 and by the assumption it should contain at least four break blocks. Since there is no break block in $[t+14, t+329]$, the four break blocks should be in $[t-466, t+13]$ and moving the rightmost break block one to the right cannot make interval $[t-466, t+13]$ infeasible. This is a contradiction.
Case 1-3: interval $[t-646, t+13]$
Interval $[t-646, t+329]$ is longer than 780. Therefore, at the worst case, the tour length is 780, it ends at $t+330$ and starts at $t-449$. So, there are at least four break blocks in $[t-646, t-450]$ and moving break block $[t-1, t+13]$ one to the right does not make interval $[t-646, t+13]$ infeasible.
Case 2: We cannot move the break block one to the right.
This means interval $[t+14, t+329]$ doe not have a break block and $t+14$ is the start of an inflexible interval . Since $t+329$ is in a flexible interval there should be an inflexible interval after. Let $t' \geq t+329$ denotes the end of the closest such inflexible interval to $t+329$. It follows that $|[t+14, t']| \geqslant 301$. Therefore by the first condition, it should have a break block. Since interval $[t, t+329]$ does not have a break block, there should be a break block in $[t+330, t']$. We shift that break block to the leftmost position. If it completely lies inside $[t, t+329]$ we are done. Otherwise, it means that there is an inflexible interval ending in $[t+315, t+329]$. At worst, it ends at $t+315$. Then we have

$$|[t, t+315]| \geq 301$$

It implies that there should be a break block inside $[t, t+315]$ by the first condition. This is a contradiction.
BB-1: The analysis is analogous to case BN-1. We just need to argue for the case that the left break block cannot move one to the right. At the worst case, the right break block overlapping $t+329$ starts at $[t+316]$ and it cannot be shifted one to the left due to an inflexible interval ending at $t+315$ and there is an inflexible interval starting at $t+14$. It follows that

$$|[t+14, t+315]| \geq 301$$

By the first condition, there should be a break block in $[t+14, t+315]$. This is a contradiction.
UB-2, UN-2, UU-2, NB-2, NN-2, NU-2:
If $[t, t+479]$ is infeasible then $[t-1, t+478]$ is also infeasible and this is a contradiction.

BU-2: If we move the interval $[t, t+479]$ one to the left, it should contain at least two break blocks with the left break block $[t-1, t+13]$ otherwise it is a contradiction. There are two possible cases: either we can move the break block one minute to the right or not. In the former case, we can make the interval feasible by moving it one to the right but it might make another interval ending at $t+13$ infeasible. There are three possible cases: either the interval $[t-316, t+13]$ of length 330 or interval $[t-466, t+13]$ of length 480 or interval $[t-646, t+13]$ of length 660 becomes infeasible. In the first case, interval $[t-316, t+479]$ is longer than 660, so, there should be at least three break blocks inside $[t-316, t+13]$. This is a contradiction. In the second case, the interval $[t-466, t+479]$ is longer than 780. So, the start of the tour, i.e., s_1, cannot be earlier than $t+479-780+1 = t-300$. Interval $[t-466, t-301]$ is entirely a break period thereby containing at least 2 break blocks which is a contradiction with $[t-466, t+13]$ getting infeasible after moving break block $[t-1, t+13]$ one to the right. We can use a similar argument for the interval $[t-646, t+13]$. So, we skip it. In the latter case, since there is one break block inside $[t+14, t+479]$, at the worst case, we have a situation in which, there is an inflexible interval starting at $t+29$. It implies that interval $[t+29, t+479]$ is infeasible. This is a contradiction since the length of this interval is equal to 451 and it should have at least 2 break blocks according to the second condition.

BB-2:
There should be one break block inside $[t, t+479]$ and one break block overlapping t and ending at $t+13$ otherwise interval $[t-1, t+478]$ is infeasible and as a result we have a contradiction. Consider the break block overlapping $t+479$. If it can be pushed fully inside $[t, t+479]$, then we are done; because then we have two break blocks inside $[t, t+479]$ and the interval becomes feasible. Otherwise, there should be an inflexible interval ending at $t'' \geq 450$. In case that $t'' \in [451, 463]$, the only break block inside $[t, t+479]$ should be the break block $[t''+1, t''+15]$ which is preventing the rightmost break block overlapping $t+479$ fully pushed inside. We consider two cases for the leftmost break block $[t-1, t+13]$
Case 1: it can move one to the right
The argument is similar to case 1 for case BN-1.
Case 2: it cannot move one to the right
Let t' and t'' be the start of leftmost inflexible interval and the end of the rightmost inflexible interval in $[t, t+479]$ respectively. Either there is an inflexible interval starting at $t+14$ or there is the break block $[t+14, t+28]$ and an inflexible interval starting at $t+29$. Therefore, $t' \leqslant t+29$. It follows from this and $t'' \geqslant 450$ that $|[t', t'']| \geqslant 301$. Therefore, based on the first condition, there is at least one break block in $[t', t'']$. If there is another break block outside of $[t', t'']$ but inside of $[t, t+479]$, we are done with having a feasible solution. Otherwise, $|[t''+1, t+479]| + |[t, t'-1]| \leqslant 28$ and $|[t', t'']| \geqslant 452$. Following the second condition, there should be at least two break block in $[t', t'']$ and thereby in $[t, t+479]$. This is a contradiction.
BN-2:
The argument is similar to the argument for case BB-2.

UU-3,UB-3,UN-3,NU-3,NN-3,NB-3:
If $[t, t+659]$ is infeasible then $[t-1, t+658]$ is also infeasible and this is a contradiction.

BU-3:
There should be three break blocks inside $[t, t+659]$ and the left break block overlapping t should start at $t-1$ otherwise interval $[t-1, t+658]$ is infeasible and as a result we have a contradiction. We consider two cases for the left break block
Case 1: it can move one to the right
In this case, one of the intervals with lengths 330, 480, or 660 ending at $t+13$, that is, one of the intervals $[t-a_k+14, t+13]$ for $k \in [3]$ might become infeasible. In all cases, interval $[t-a_k+14, t+659]$ is longer than 780. So, at the worst case, the tour starts no earlier than $t-120$. As per our assumption that the intervals before and after the tour are entirely breaks, there are at least 5 break blocks in all $k \in [3]$ intervals $[t-a_k+14, t-120]$. As a result, there are at least 5 break blocks inside all $k \in [3]$ intervals $[t-a_k+14, t+13]$. This is a contradiction and moving one break block outside of the three intervals ending at $t+13$ does not make the infeasible.

Case 2: it cannot move one to the right
We consider three cases:
Case 2-1: There is an inflexible interval starting at $t + 14$. Let t' be the end of the rightmost inflexible interval overlapping $t + 659$. So $|[t + 14, t']| \geqslant 631$. It follows from the fourth condition that we have four break blocks inside $[t + 14, t']$ thereby four break blocks inside $[t, t + 659]$. This is a contradiction.
Case 2-2: There is an inflexible interval starting at $t + 29$ and one break block from $t + 14$ to $t + 28$. So $|[t + 29, t']| \geqslant 631$. It follows from the fourth condition that we have four break blocks inside $[t + 29, t']$ and thereby four break blocks inside $[t, t + 659]$. This is a contradiction.
Case 2-3: There is an inflexible interval starting at $t + 44$ and two break blocks from $t + 14$ to $t + 43$ So $|[t + 44, t + 659]| \geqslant 480$. It follows from the second condition that we have two breaks inside $[t + 44, t + 659]$ and thereby four break blocks inside $[t, t + 659]$. This is a contradiction.
Case 2-4: There is an inflexible interval starting at $t + 59$ and three break blocks from $t + 14$ to $t + 58$, So $|[t + 59, t + 659]| \geqslant 301$. It follows from the first condition that we have one break block inside $[t + 59, t + 659]$ and thereby four break blocks inside $[t, t + 659]$. This is a contradiction.

BB-3:
There should be three break blocks inside $[t, t + 659]$ and one break block overlapping t and starting at $t - 1$. Otherwise interval $[t - 1, t + 659]$ is infeasible and as a result we have a contradiction. We assume the right break block overlapping $t + 659$ cannot be shifted to the left so that it completely is inside interval $[t, t + 659]$. We consider two cases for the left break block:
Case 1: it can move one to the right
Analogous to Case 1 for BU-3.
Case 2: it cannot move one to the right
According to the assumption, there are three break blocks inside $[t + 14, t + 659]$. Let t' denote the start of leftmost inflexible interval and t'' the end of the rightmost inflexible interval inside $[t, t + 659]$. Since neither the leftmost nor the rightmost break block can be pushed inside, we have multiple cases depending on how these three blocks are distributed among intervals $[t, t' - 1]$, $[t', t'']$, and $[t'' + 1, 659]$. We consider four cases depending on how many break blocks are inside $[t', t'']$ starting from 3 down to 0. In the first case, $|[t, t' - 1]| + |[t'' + 1, 659]|$ cannot be bigger than $2(\delta - 1) = 28$ because both intervals $[t, t' - 1]$ and $[t'' + 1, 659]$ are all break given the rightmost break overlapping $t + 659$ fully pushed to the left. Therefore, $|[t', t'']| \geqslant 632$. Following the fourth condition, there should be four break blocks inside $[t', t'']$ thereby having four break blocks in the interval $[t, t + 659]$ leading to a contradiction. In the second case in which we have two break blocks inside $[t', t'']$ and one break block either in $[t, t' - 1]$ or in $[t'' + 1, 659]$, so $|[t, t' - 1]| + |[t'' + 1, 659]|$ cannot be bigger than $2(\delta - 1) + \delta = 43$. Therefore, $|[t', t'']| \geqslant 616$. This and the third condition lead to a contradiction again. Other cases are analogous. So we skip them.

BN-3:
There should be three break blocks inside $[t, t + 659]$ and one break block overlapping t and starting at $t - 1$. Otherwise interval $[t - 1, t + 659]$ is infeasible and as a result we have a contradiction.

If there is a break block inside the flexible interval overlapping $t + 659$ which can be pushed partially to the left inside interval $[t, t + 659]$ then this case will turn into case $BB - 3$. Otherwise, there is no break block to the right of $[t, t + 659]$ and inside the flexible interval overlapping $t + 659$. We consider two cases for the leftmost break block overlapping t:
Case 1: it can move one to the right
Analogous to Case 1 for BU-3.
Case 2: it cannot move one to the right
According to the assumption, there are three break blocks inside $[t + 14, t + 659]$. Let t' denote the start of leftmost inflexible interval and t'' the end of the leftmost inflexible interval outside $[t, t + 659]$. Since the

leftmost break block cannot be pushed fully inside, we have four cases depending on how these three blocks are distributed between intervals $[t, t'-1]$, $[t', 659]$. We start from 3 going down to 0. In the first case, $t' = t + 14$ and thereby $|t', t''| \geqslant 631$. Following the fourth condition, there should be four break blocks inside $[t', t'']$ and thereby one break block inside $[t + 659, t'']$, this is a contradiction to assuming there is no break block inside the inflexible interval overlapping $t + 659$. In the second case, $t' = t + 29$ and thereby $|t', t''| \geqslant 616$. Following the third condition, we have three break blocks inside $[t', t'']$ and thereby one break block inside $[t + 659, t'']$. This is a contradiction again. The argument for the other cases is analogous.